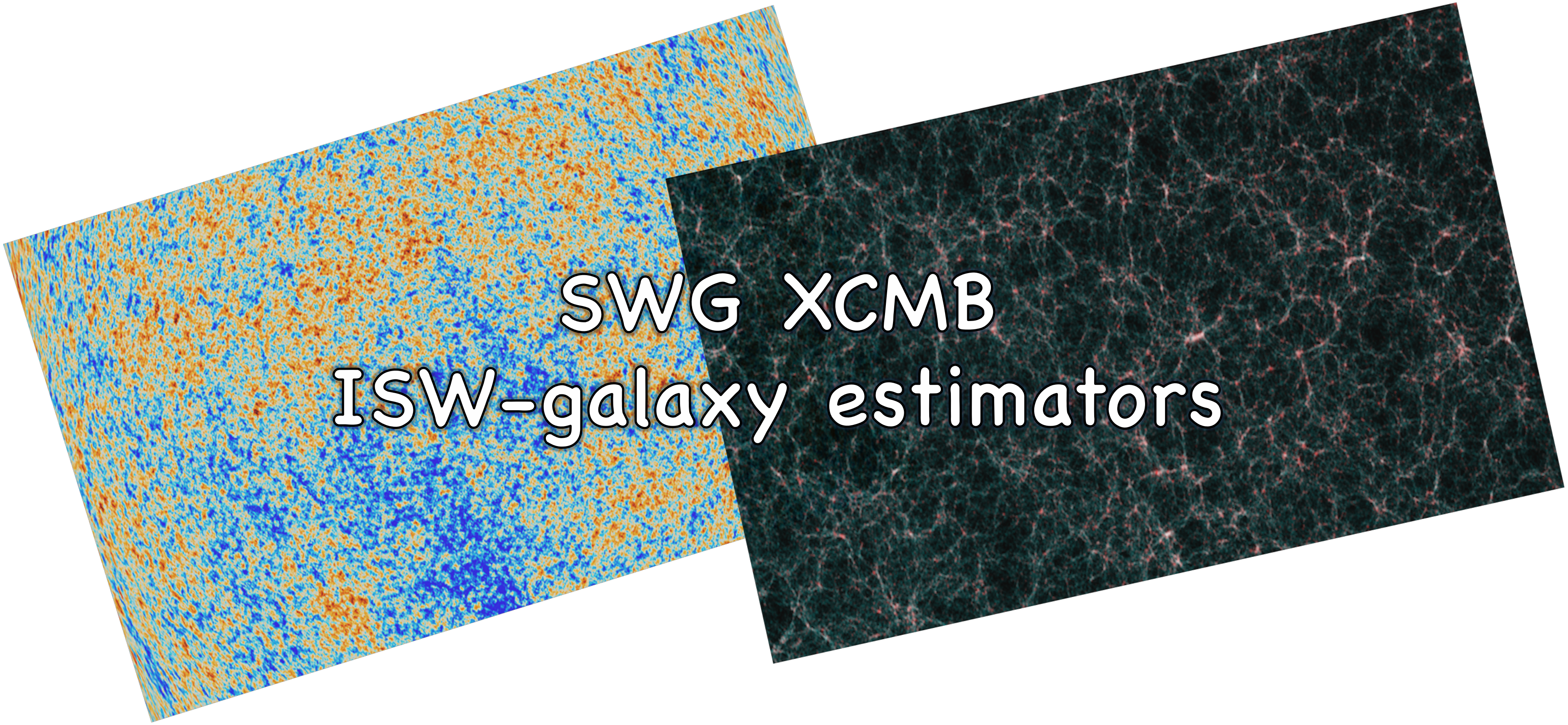
SWG XCMB
ISW-galaxy estimators

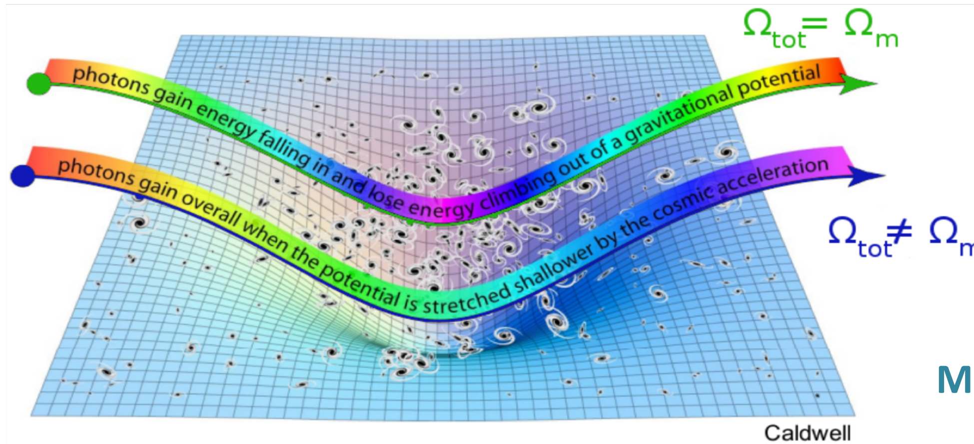


Marina Migliaccio (Roma UniTOV) and Alessandro Gruppuso (INAF-OAS Bo)

for the WP estimators

Euclid CMBX SWG Meeting, 19th March 2020

Measuring the Integrated Sachs-Wolfe effect
CMB x galaxy cross-correlation

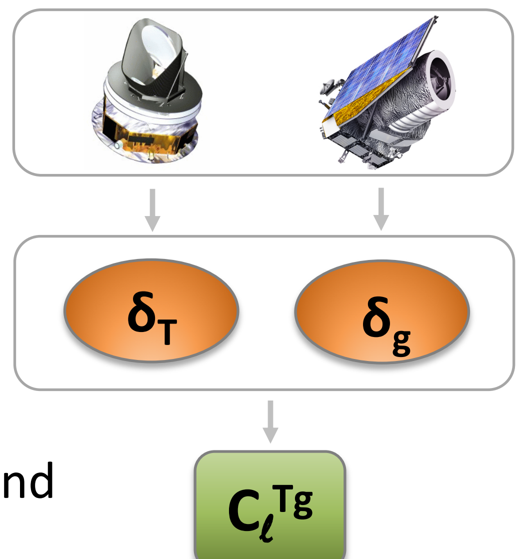


Dark Energy
Modified Gravity

$$\delta_T(\mathbf{n}_2) = \frac{T(\mathbf{n}_2) - \bar{T}}{\bar{T}} = -2\int e^{-\tau(z)} \frac{d\Phi}{dz}(\mathbf{n}_2, z)dz$$

$$\delta_g(\mathbf{n}_1) = \frac{N_g(\mathbf{n}_1) - \bar{N}_g}{\bar{N}_g} = \int b(z) \frac{dN}{dz}(z)\delta_m(\mathbf{n}_1, z)dz$$

$$C^{\mathrm{Tg}}(\vartheta) = \left\langle \delta_T(\mathbf{n}_1)\delta_g(\mathbf{n}_2) \right\rangle$$



Estimators: cross-correlation in real space, wavelet and needlet space, Harmonic Space

Summary of activities

1) **Zero-th order simulations**
   - Developed framework to generate correlated maps of CMB T and Galaxy counts.
   - Fully extended to tomographic and multi-survey analyses.

2) **Realistic masks**
   - Tools for mask generation

3) **Pseudo-Cl estimator**
   - Developed a code which has been validated on zero-th order simulations.
   - Extended and tested also for tomographic analyses.
   - Analytic covariance approximation developed and tested on simulations.

4) **Quadratic Maximum Likelihood estimator**
   - Developed a QML code for $\begin{pmatrix} TT & TG \\ TG & GG \end{pmatrix}$ which has been validated on zero-th order simulations. Tested for one and two redshift bins at low resolution (Nside 16).
   - Developed a new formalism for TG-only spectrum that is computationally less expensive. Explored different parallelisation strategy. The code has been validated on simulations for a single redshift bin (Nside 32).

5) **Comparison of estimators**
   - Preliminary results for a single redshift bin

Signal-to-Noise

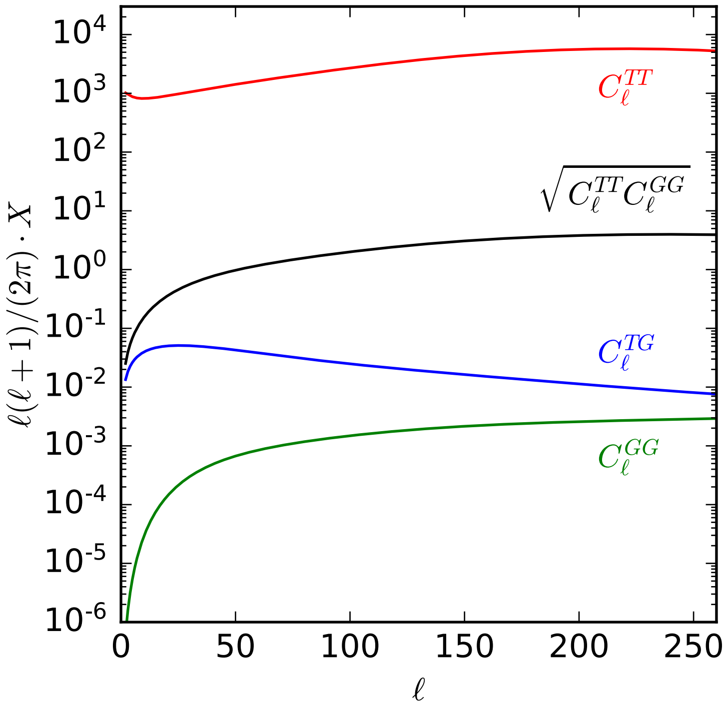
Assuming Planck 2018 cosmology and considering the whole photometric survey

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)^2 = \sum_{\ell=2}^{\ell_{\max}} \frac{(\mathrm{C}_\ell^{\mathrm{Tg}})^2}{(\Delta \mathrm{C}_\ell^{\mathrm{Tg}})^2}$$

$$= \sum_{\ell=2}^{\ell_{\max}} (2\ell+1) \mathrm{f}_{\mathrm{sky}} \frac{(\mathrm{C}_\ell^{\mathrm{Tg}})^2}{(\mathrm{C}_\ell^{\mathrm{Tg}})^2 + (\mathrm{C}_\ell^{\mathrm{TT}} + \mathrm{N}_\ell^{\mathrm{TT}})(\mathrm{C}_\ell^{\mathrm{gg}} + \mathrm{N}_\ell^{\mathrm{gg}})}$$

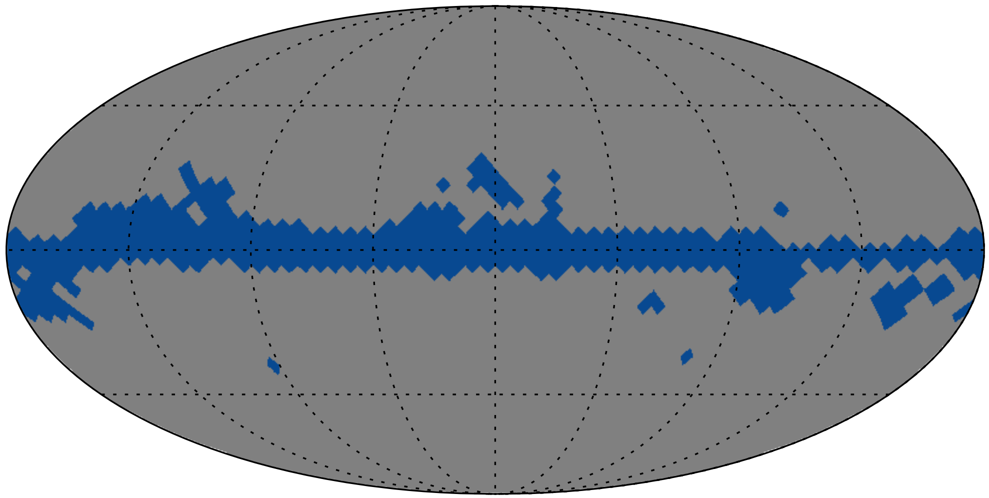
	QML $\mathrm{N_{side}}$ 32	QML $\mathrm{N_{side}}$ 64	PCL $\mathrm{N_{side}}$ 128
$\ell_{\max}$	96	192	256
S/N	3.77	3.83	3.84

$\mathrm{f_{sky}} = 0.37$ $\quad \mathrm{N}_\ell^{TT}, \mathrm{N}_\ell^{gg} \approx 0$

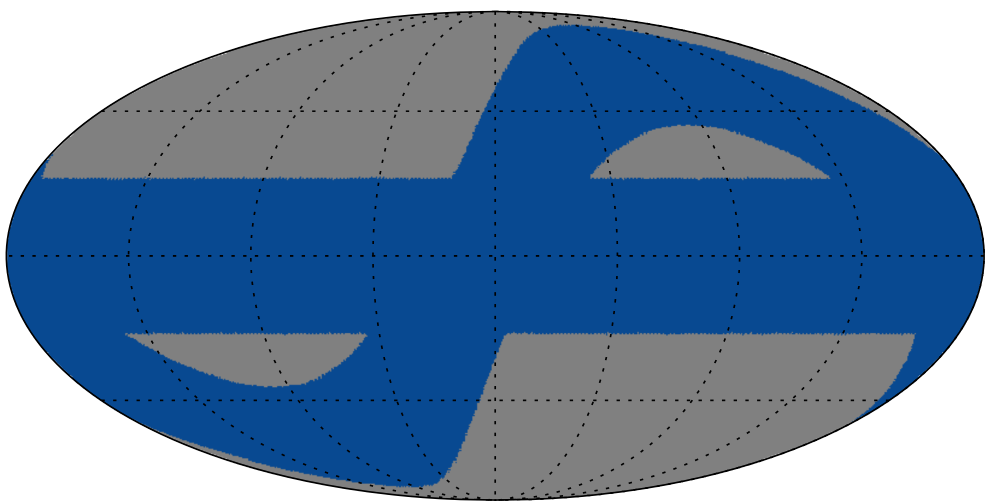


Simulations and Masks

Sets of 1000 T and G correlated simulations at Nside 32 and 128
Assuming Planck 2018 cosmology and considering the whole photometric survey



T mask
fsky = 0.86
(Commander 2018)



G mask
fsky = 0.37
(Red Book)

To be updated with
wide survey mask
used by OU L3

Pseudo-Cl estimator: Analytic Covariance Matrix

$$\langle \Delta\tilde{C}_{\ell_1}^{TG_i} \Delta\tilde{C}_{\ell_2}^{TG_j} \rangle = \frac{1}{2\ell_2+1}\Big[M^{(2)}_{\ell_1\ell_2}(W^{TT,G_iG_j})\sqrt{C_{\ell_1}^{TT}C_{\ell_2}^{TT}(C_{\ell_1}^{G_iG_j}+N_{\ell_1}^{G_iG_j})(C_{\ell_2}^{G_iG_j}+N_{\ell_2}^{G_iG_j})}$$
$$+ M^{(2)}_{\ell_1\ell_2}(W^{TG_i,TG_j})\sqrt{C_{\ell_1}^{TG_i}C_{\ell_1}^{TG_j}C_{\ell_2}^{TG_i}C_{\ell_2}^{TG_j}}\Big]$$

$$M^{(2)}_{\ell_1\ell_2}(W^{AB,CD}) = \frac{(2\ell_2+1)}{4\pi}\sum_{\ell_3}(2\ell_3+1)W_{\ell_3}^{AB,CD}\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^2$$

$$W_\ell^{AB,CD} = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} w_{\ell m}^{AB} w_{\ell m}^{CD*}$$

with $w_{\ell m}^{AB}$ spherical harmonic coefficients of the product of the masks $m^{XY} = m^X \cdot m^Y$.
The shot noise $N_\ell^{G_iG_j} = 1/n$, if $i=j$, with n the number of observed galaxies per steradian, and $N_\ell^{G_iG_j} = 0$ otherwise.

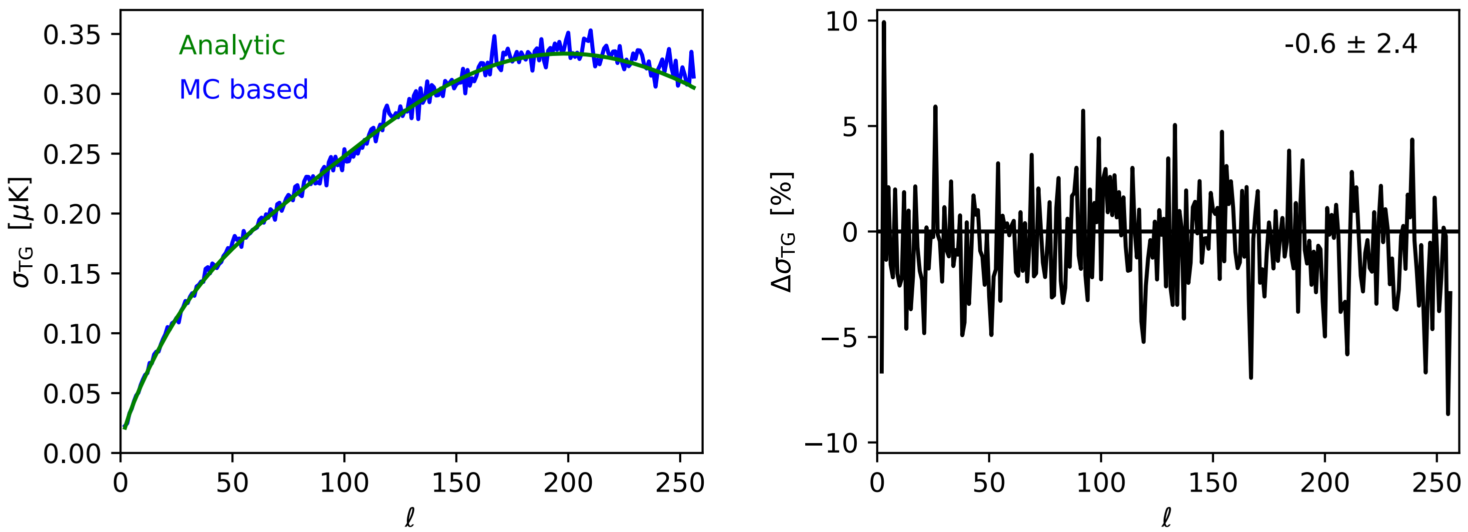
$$\mathrm{Cov}_{\ell\ell'} = \left(\mathcal{M}_{\ell\ell_1}^{TG_i}\right)^{-1}\langle \Delta\tilde{C}_{\ell_1}^{TG_i} \Delta\tilde{C}_{\ell_2}^{TG_j} \rangle \left(\mathcal{M}_{\ell_2\ell'}^{TG_j}\right)^{-1}$$

$$\mathcal{M}^{(2)}_{\ell_1\ell_2} = \frac{(2\ell_2+1)}{4\pi}\sum_{\ell_3}(2\ell_3+1)W_{\ell_3}^{AB}\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^2$$

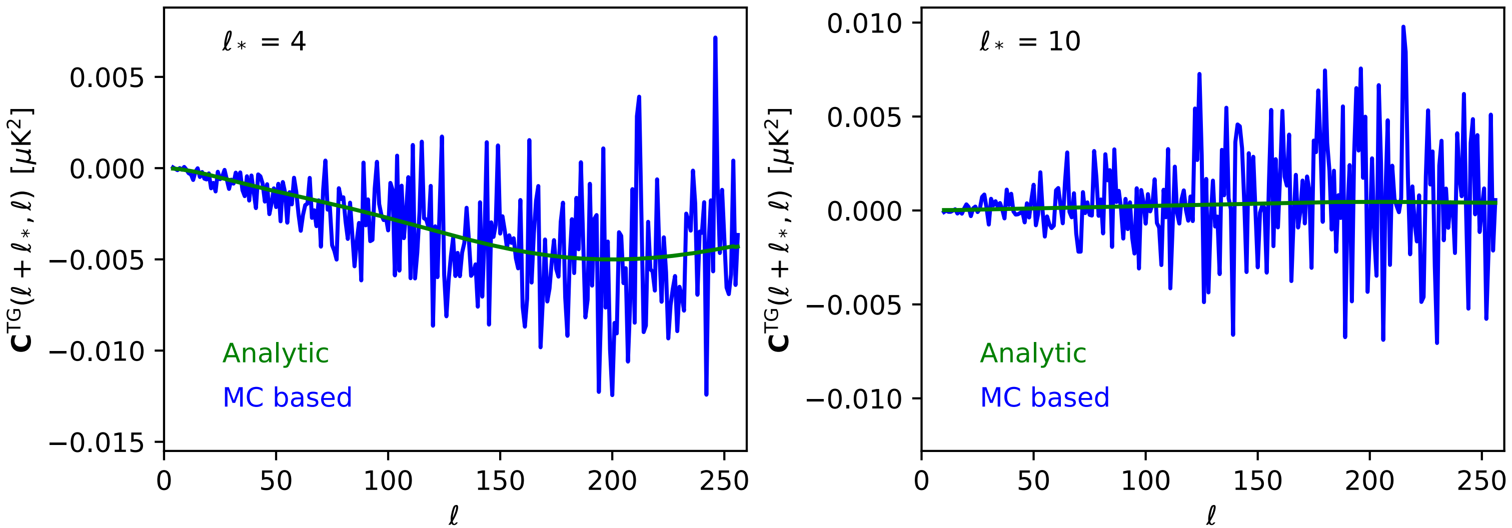
including the effect of beams and pixel window functions.

Pseudo-Cl estimator: Covariance Matrix

Error Bars



Off-diagonal terms



QML estimator

Example: case of a CMB map and one Galaxy survey/redshift bin

QML algebra

$$\begin{pmatrix} \hat{C}_\ell^{TT} \\ \hat{C}_\ell^{TG} \\ \hat{C}_\ell^{GG} \end{pmatrix} = F_{\ell\ell'}^{-1} \begin{pmatrix} y_{\ell'}^{TT} \\ y_{\ell'}^{TG} \\ y_{\ell'}^{GG} \end{pmatrix}$$

$$x = (x_{CMB}, x_G)$$

and the covariance matrix is

$$C = \langle x x^t \rangle = \begin{pmatrix} C_{CMB} & C_{TG} \\ C_{TG}^t & C_G \end{pmatrix}$$

and the Fisher matrix is

$$F_{\ell\ell'} = \begin{pmatrix} \mathrm{TT\,TT} & \mathrm{TT\,TG} & \mathrm{TT\,GG} \\ \mathrm{TG\,TT} & \mathrm{TG\,TG} & \mathrm{TG\,GG} \\ \mathrm{GG\,TT} & \mathrm{GG\,TG} & \mathrm{GG\,GG} \end{pmatrix}$$

where each entry of the matrix stands for

$$F_{\ell\ell'}^{XY} = \frac{1}{2} Tr \left[C^{-1} P_\ell^X C^{-1} P_{\ell'}^Y \right]$$

$$X, Y = TT, TG, GG$$

QML estimator

Example: case of a CMB map and one Galaxy survey/redshift bin

QML algebra

$$\begin{pmatrix} \hat{C}_\ell^{TT} \\ \hat{C}_\ell^{TG} \\ \hat{C}_\ell^{GG} \end{pmatrix} = F_{\ell\ell'}^{-1} \begin{pmatrix} y_{\ell'}^{TT} \\ y_{\ell'}^{TG} \\ y_{\ell'}^{GG} \end{pmatrix}$$

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and the Fisher matrix is

$$F_{\ell\ell'} = \begin{pmatrix} \mathrm{TT\,TT} & \mathrm{TT\,TG} & \mathrm{TT\,GG} \\ \mathrm{TG\,TT} & \mathrm{TG\,TG} & \mathrm{TG\,GG} \\ \mathrm{GG\,TT} & \mathrm{GG\,TG} & \mathrm{GG\,GG} \end{pmatrix}$$

where each entry of the matrix stands for

For our masked case

$$F_{\ell\ell'}^{XY} = \frac{1}{2} Tr\left[C^{-1} P_\ell^X C^{-1} P_{\ell'}^Y\right]$$

$$X, Y = TT, TG, GG$$

0.1 GB at $N_{side} = 16$

1.9 GB at $N_{side} = 32$

30.6 GB at $N_{side} = 64$

QML: Disentangling TG

Example: case of a CMB map and one Galaxy survey/redshift bin

THREE INDEPENDENT QMLS

$$\hat{C}_\ell^{TT} = \left(F_{\ell\ell'}^{TT,TT}\right)^{-1} y_{\ell'}^{TT}$$

$$\hat{C}_\ell^{TG} = \left(F_{\ell\ell'}^{TG,TG}\right)^{-1} y_{\ell'}^{TG}$$

$$\hat{C}_\ell^{GG} = \left(F_{\ell\ell'}^{GG,GG}\right)^{-1} y_{\ell'}^{GG}$$

$$x = (x_{CMB}, x_G)$$

and the covariance matrix is

$$C = \langle x x^t \rangle = \begin{pmatrix} C_{CMB} & C_{CMB\,G} \\ C_{G\,CMB} & C_G \end{pmatrix}$$

$$C_\ell^{fid,TG} = 0$$

and the Fisher matrix is

$$F_{\ell\ell'} = \begin{pmatrix} TT\,TT & TT\,TG & TT\,GG \\ TG\,TT & TG\,TG & TG\,GG \\ GG\,TT & GG\,TG & GG\,GG \end{pmatrix}$$

where each entry of the matrix stands for

$$F_{\ell\ell'}^{XY} = \frac{1}{2} Tr\left[C^{-1} P_\ell^X C^{-1} P_{\ell'}^Y\right]$$

$$X, Y = TT, TG, GG$$

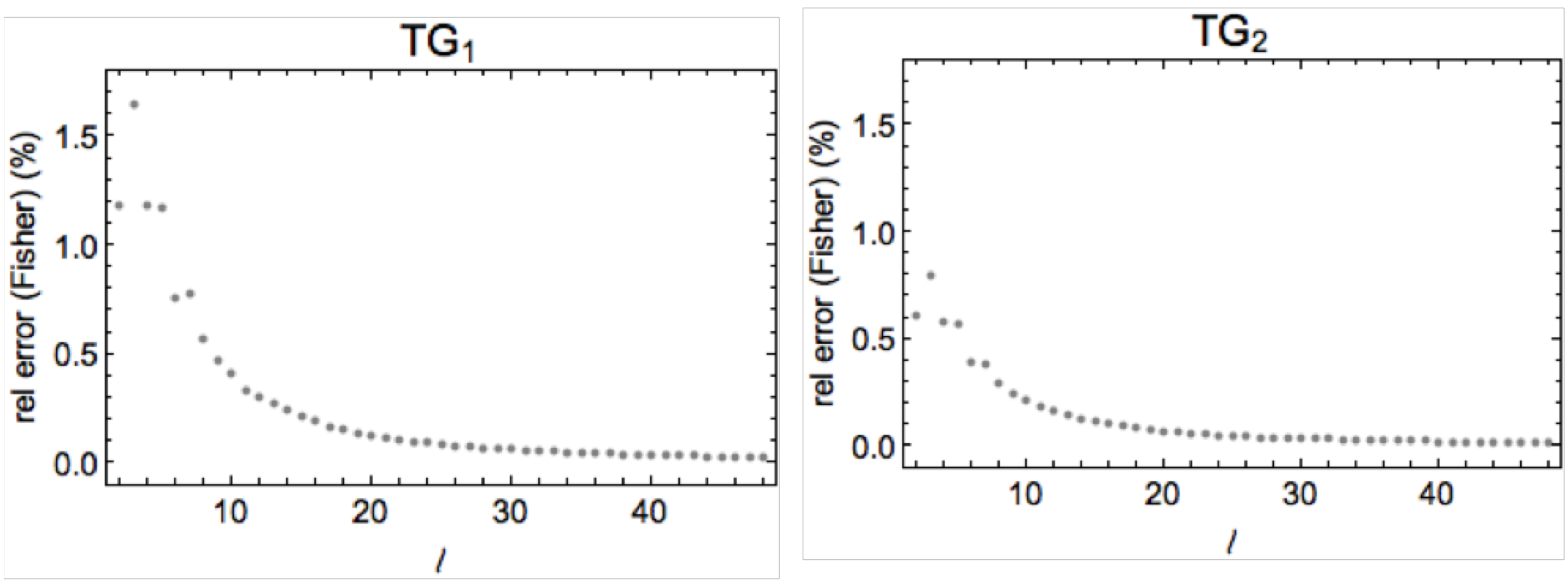
QML: Disentangling TG

when we consider or not fiducial cross spectra

→ Estimates remain unbiased

→ Small impact on Fisher uncertainties

Nside=16



When the fiducial is not exactly the one used generate the MC, the QML is not exactly optimal anymore. This quantifies the increase of the error in percentage for each multipole when we use this simplification.

TG-only QML estimator

$$\hat{C}_\ell^{TG} = F^{-1}_{\ell\ell'} y_{\ell'}$$

$$F_{\ell\ell'} = Tr\left[C^{-1}_{TT} P_\ell C^{-1}_{GG} (P_{\ell'})^t\right] \quad y_\ell = m_T^t C^{-1}_{TT} P_\ell C^{-1}_{GG} m_G$$

TG-only QML estimator

For our masked case

$$\hat{C}_\ell^{TG} = F^{-1}_{\ell\ell'} y_{\ell'}$$

0.2GB at nside 32

6.6GB at nside 64

$$F_{\ell\ell'} = Tr\left[C^{-1}_{TT} P_\ell C^{-1}_{GG} (P_{\ell'})^t\right] \quad y_\ell = m_T^t C^{-1}_{TT} P_\ell C^{-1}_{GG} m_G$$

3GB at nside 64

0.2GB at nside 32

14GB at nside 64

0.9GB at nside 32

Fewer elements to be computed
and each of them is lighter

We were able to compute the estimates at Nside=32 in 20 min using 960 cores at NERSC.

TG-only QML estimator: Validation

Nside = 32

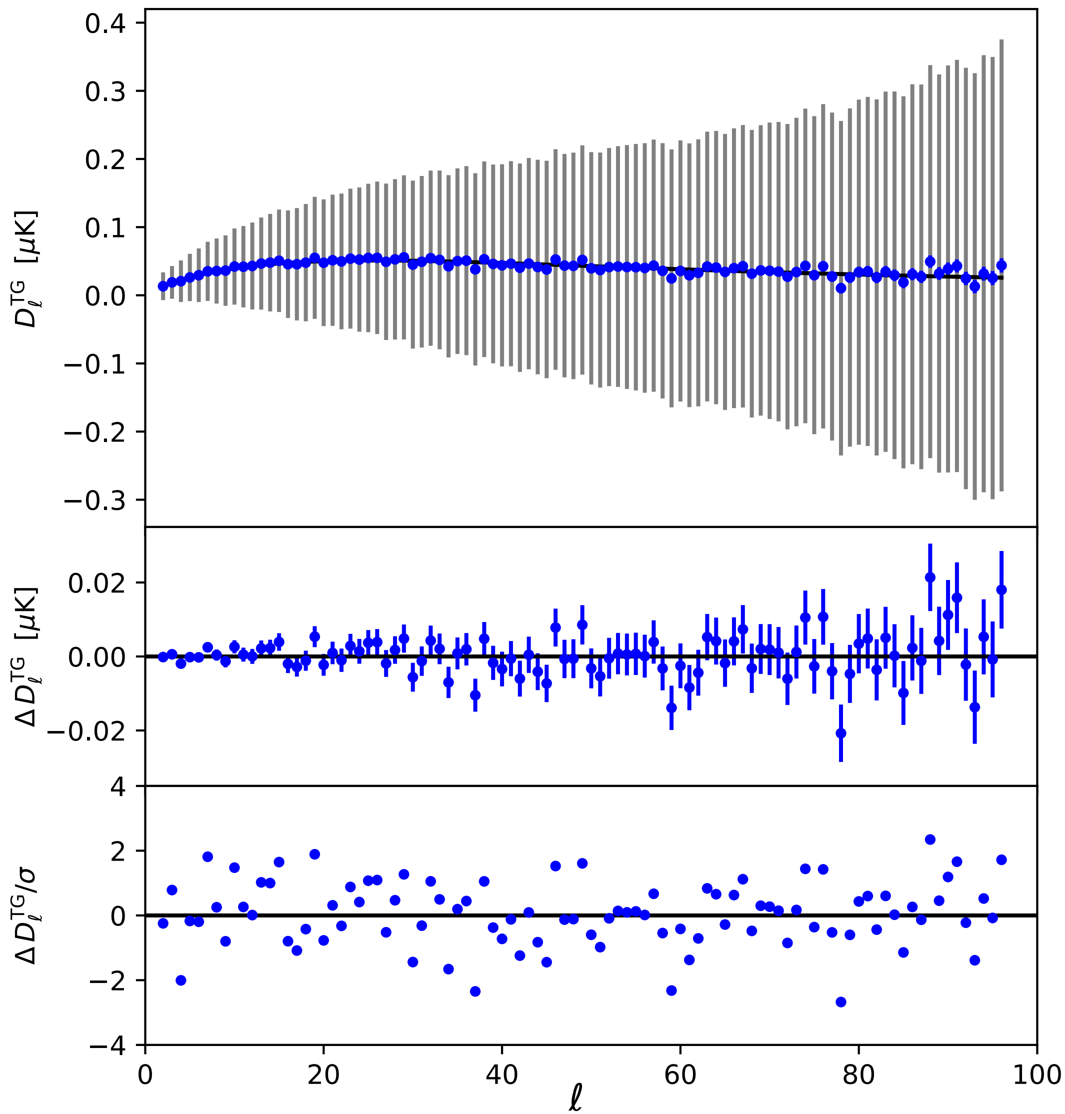
Mean of 1000 sims

Error of the mean

$\sigma^{TG} / \sqrt{N_{sim}}$

Error Bars

$\sigma^{TG} = (F_\ell)^{-1}$

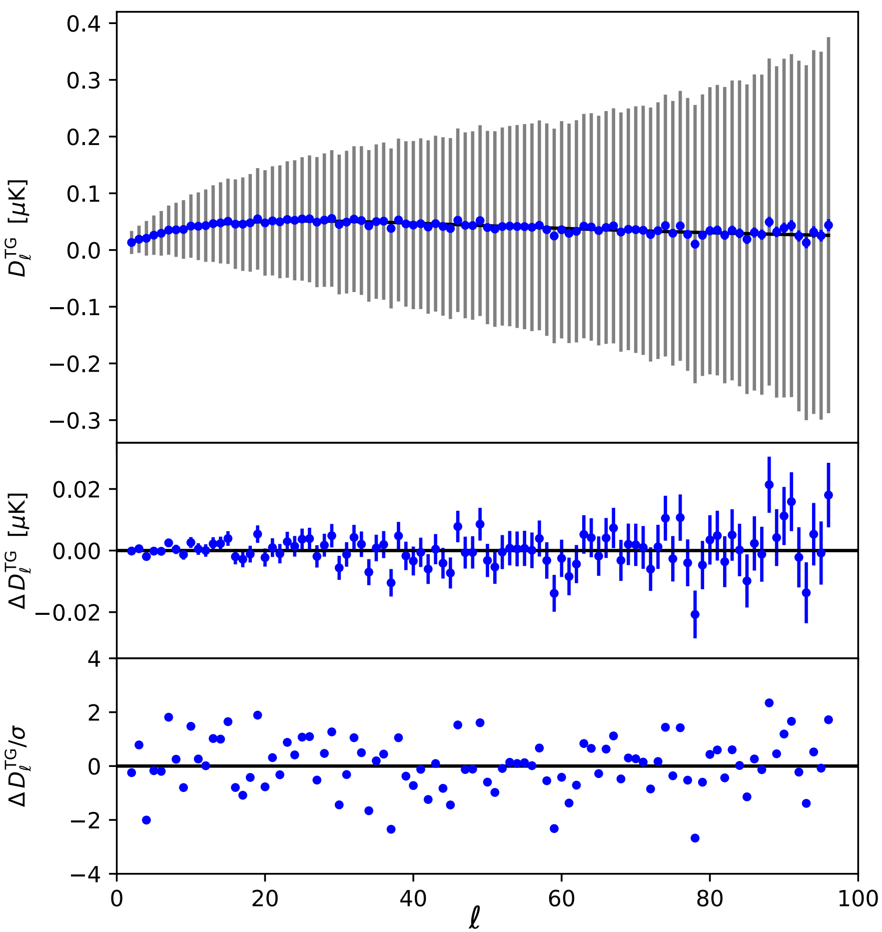


Comparison of estimators

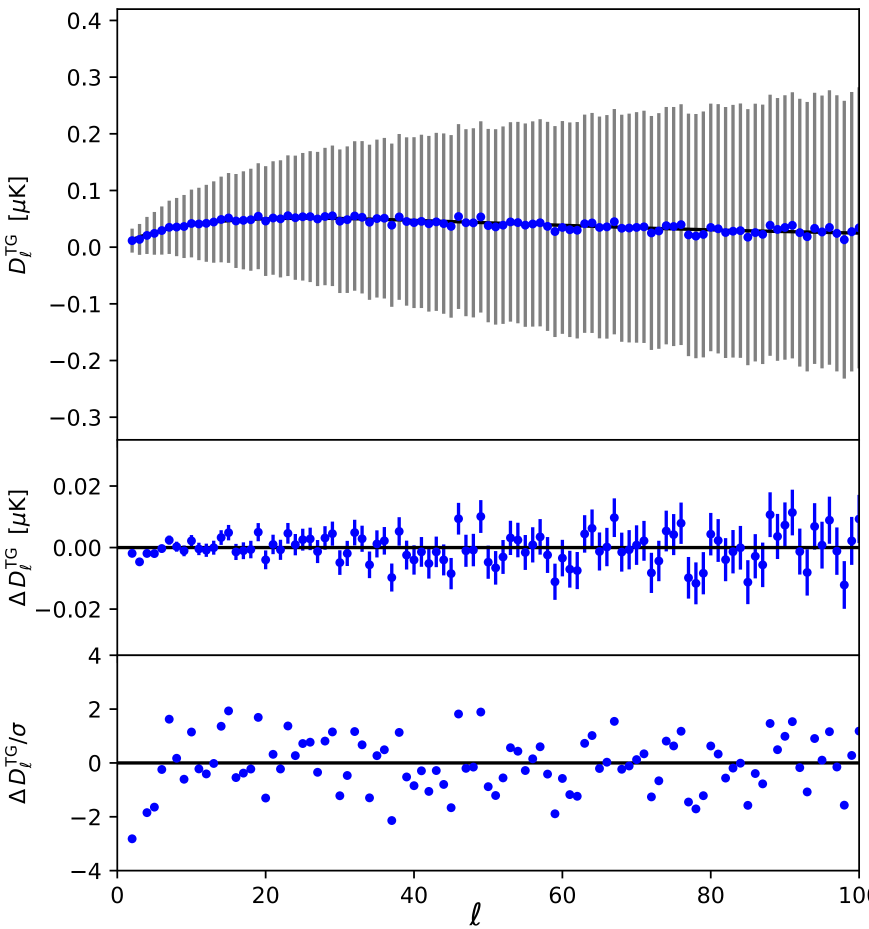
Nside = 32

Nside = 128

TG-only QML



Pseudo-Cl

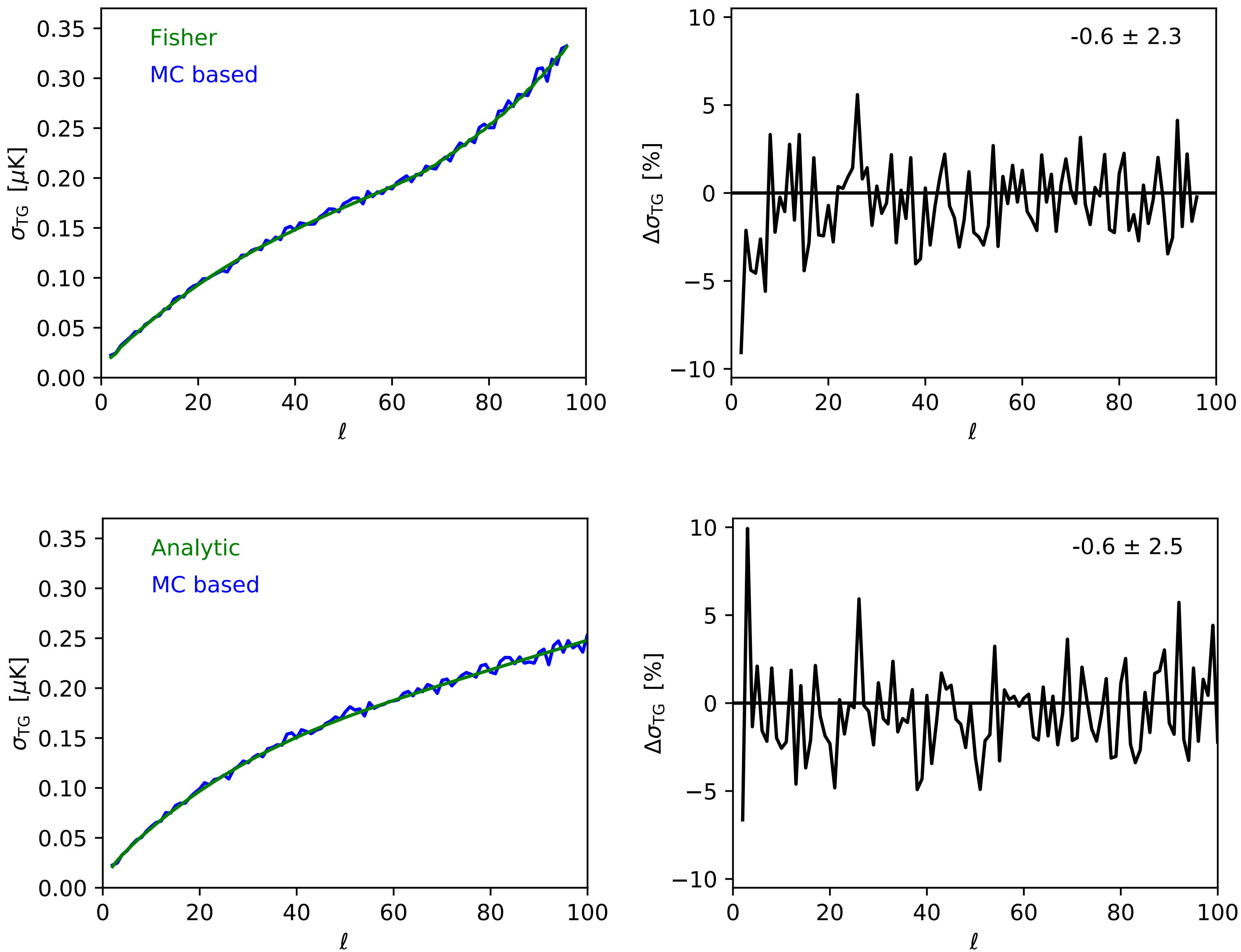


Zoom on the range of multipoles covered by both estimators

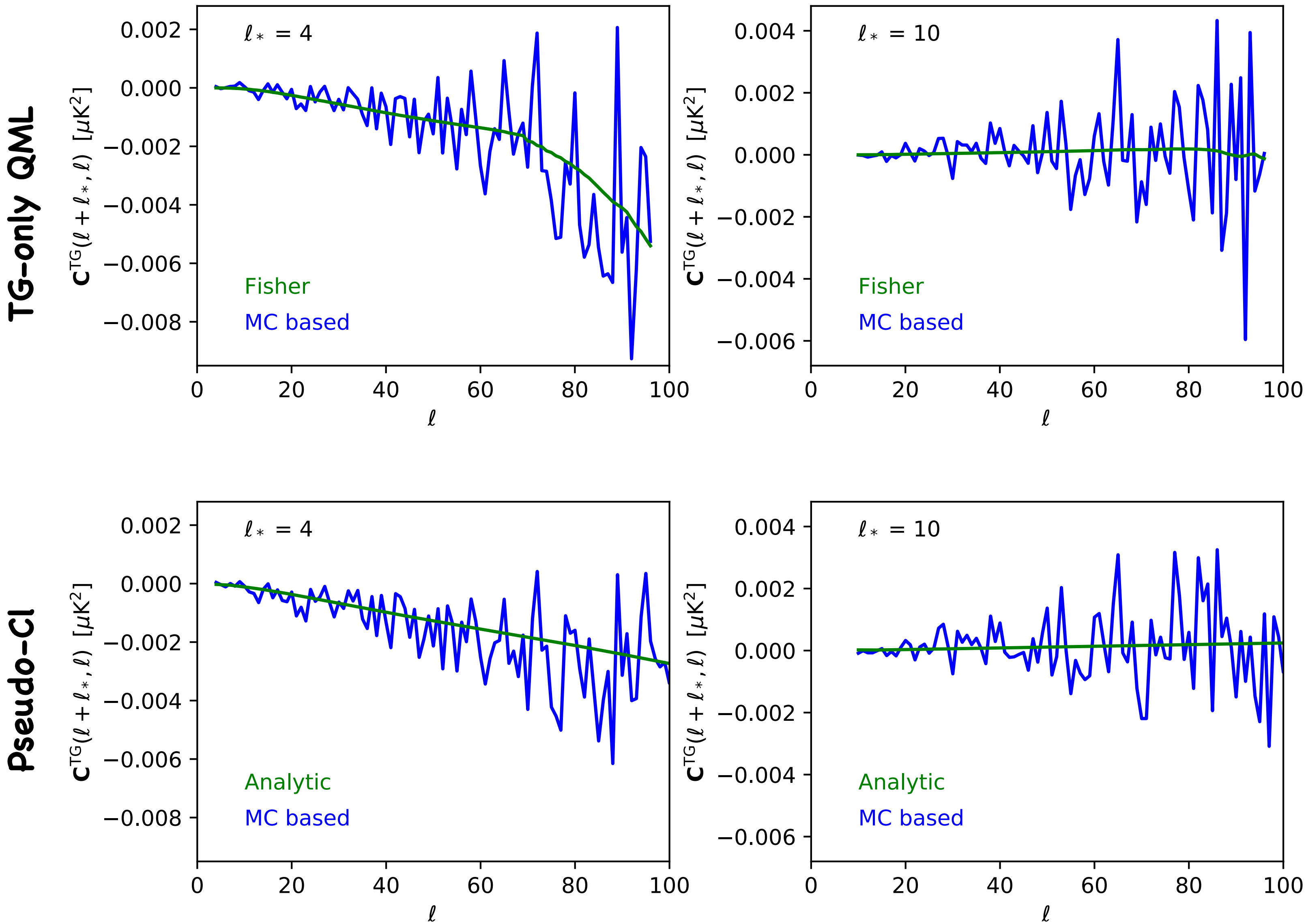
Comparison of Covariance Matrices
Error Bars

TG-only QML

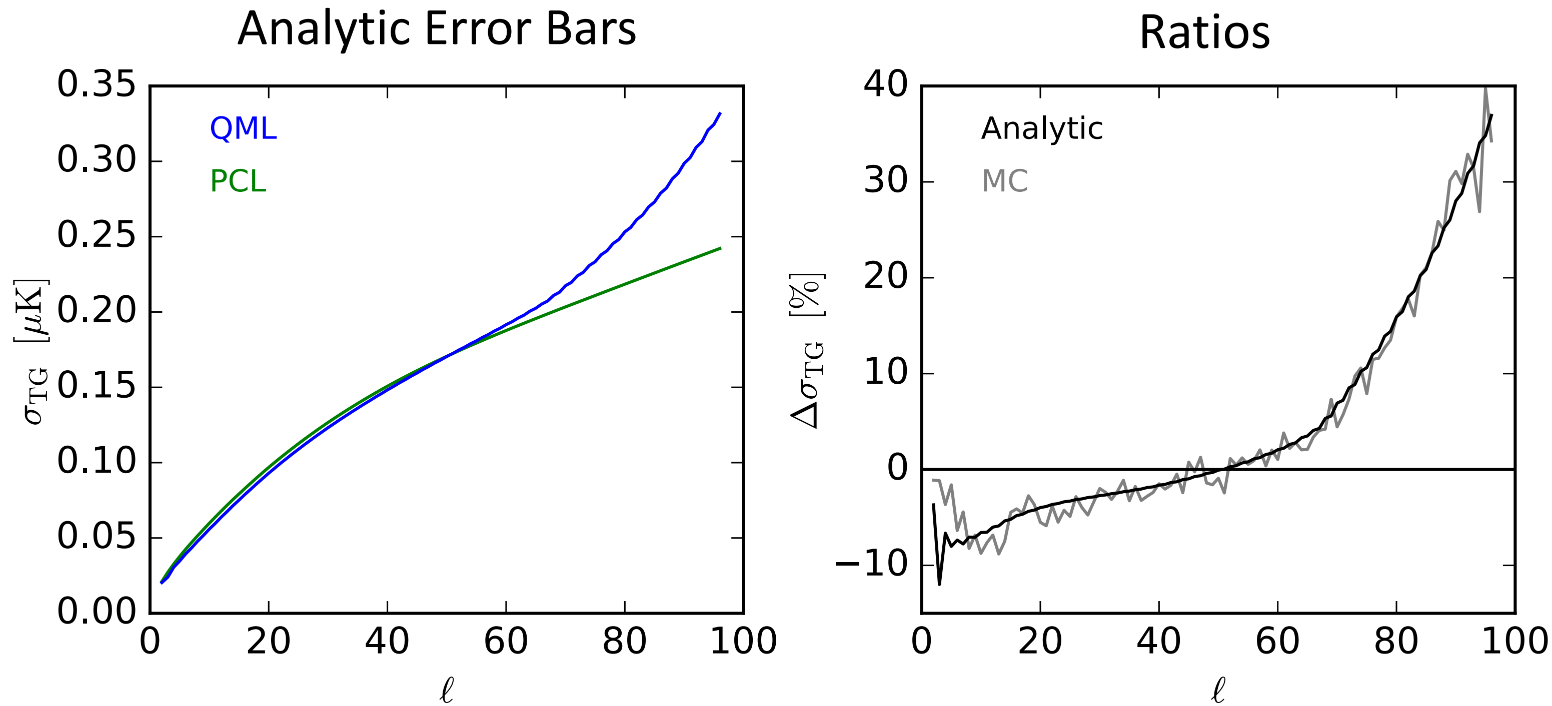
Pseudo-Cl



Comparison of Covariance Matrices
Off-diagonal Terms



Comparison of Covariance Matrices
Error Bars



QML has from 2 to 10% smaller error bars in the range of multipoles where there is the maximum of TG for the full photometric survey.

$$\mathrm{S}/N = \left(\sum_{\ell,\ell'}^{\ell_{max}} \mathrm{C}_{\ell}^{\mathrm{T}G} \mathrm{C}ov_{\ell\ell'}^{-1} \mathrm{C}_{\ell'}^{\mathrm{T}G} \right)^{1/2}$$

PCL $\ell_{\max}$ = 256 S/N = 3.85

QML $\ell_{\max}$ = 96 S/N = 3.88

Ongoing activities

Pseudo-Cl estimator

- Estimator and analytic covariance matrix fully validated on simulations, including tomography
- Ready to: interfacing to N-body sims and mock catalogues from the EC and SWG; adding CMB polarization and lensing information
- Can be used also for CMB lensing x GC

QML estimator

- Ongoing effort to reach even higher resolution (Nside 64)
- Development of a covariance matrix for tomographic correlated redshift bins.

Further comparison of the two estimators: ℓ by ℓ on the same simulation; extension to a tomographic case study.

Interface to the likelihood pipeline to assess the performance of the estimators at cosmological parameter level.